Universität Konstanz

Fachbereich
Mathematik und Statistik
Schwerpunkt
Reelle Geometrie und Algebra

Einladung

Im Oberseminar *Modelltheorie* hält

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am **Montag, 16.01.2017**, einen Vortrag zum Thema:

Real exponentiation, Schanuel's conjecture and the generalised Lindemann - Weierstrass theorem

Der Vortrag findet um **15:15 Uhr** in **F426** statt.
Alle Interessenten sind herzlich eingeladen.

Sebastian Gruler

Koordinator Oberseminar

Abstract: In 1996, Macintyre and Wilkie achieved several results about the theory of the ordered field of real numbers with exponentiation. Wilkie managed to prove its model completeness in [6]. Based on this result, in [3] Macintyre and Wilkie developed a recursive subtheory of the theory of real exponentiation and showed that this subtheory axiomatizes the theory of real exponentiation under the assumption of a famous conjecture of transcendental number theory. It is known as Schanuel's conjecture and was first mentioned in the literature by Schanuel's doctoral supervisor Lang in [1]. The conjecture states that for any over $\mathbb{Q}$ linearly independent numbers $a_1, \ldots, a_m$ the transcendence degree of $a_1, \ldots, a_m, e^{a_1}, \ldots, e^{a_m}$ over $\mathbb{Q}$ is at least m. Its significance lies not only in the fact that it would prove the decidability of the theory of real exponentiation as described above, but also in its ability to deduce other unknown algebraical properties such as the algebraic independence of e and π. The algebraic nature of these numbers had been studied long before the appearance of Schanuel's conjecture. Already in 1882, Lindemann proved that e^a is transcendental for every non-zero algebraic number a, from which he deduced the transcendence of π. He published his results in [2], where he also mentioned a more general statement without proof, namely that for arbitrary distinct algebraic numbers $a_1, \ldots, a_m$, the numbers $e^{a_1}, \ldots, e^{a_m}$ are linearly independent over the algebraic numbers. Some years later, Weierstrass gave a detailed proof of this theorem in [5]. It is therefore known as the Lindemann-Weierstrass theorem. In my talk I will present a proof of the Lindemann-Weierstrass theorem from 1956, given by Niven in [4]. In the end, I am going to point out some yet unproven assumptions of transcendental number theory that could be proved using Schanuel's conjecture, but do not follow from the generalized Lindemann-Weierstrass theorem.

References

[1] S. Lang. *Introduction to transcendental numbers*. Addison-Wesley series in mathematics. Addison-Wesley Pub. Co., 1966.

[2] F. Lindemann. Über die Zahl π. *Math. Ann.*, 20(2):213–225, 1882.

[3] Angus Macintyre and A. J. Wilkie. On the decidability of the real exponential field. In *Kreiseliana*, pages 441–467. A K Peters, Wellesley, MA, 1996.

[4] Ivan Niven. *Irrational numbers*. The Carus Mathematical Monographs, No. 11. The Mathematical Association of America. Distributed by John Wiley and Sons, Inc., New York, N.Y., 1956.

[5] K. Weierstrass. Zu Lindemann's Abhandlung: „Über die Ludolph'sche Zahl". pages 1067–1086, 1885.

[6] A. J. Wilkie. Model completeness results for expansions of the ordered field of real numbers by restricted Pfaffian functions and the exponential function. *J. Amer. Math. Soc.*, 9(4):1051–1094, 1996.