Universität Konstanz

Fachbereich
Mathematik und Statistik

Prof. Dr. Salma Kuhlmann
Katharina Dupont
Gabriel Lehéricy

Real Algebraic Geometry II
Exercise Sheet 9

Exercise 1
Let k be a field and G an ordered abelian group. Let $\mathbb{K} = k((G))$ be the field of generalized power series. Let $i := \sqrt{-1}$.
Show that $k(i)((G)) \cong k((G))(i)$.

Exercise 2
Let K be a real closed field. Let v be the natural valuation on K, $G := v(K^\times)$ its value group and $k = \overline{K}$ its residue field. Show that

(a) G is divisible,

(b) k is real closed.

Exercise 3
(a) Show that $\text{card}\left(\mathbb{Q}^{rc}((\mathbb{Q}))\right) = 2^{\aleph_0}$.

(b) Find a countable non-archimedean subfield of $\mathbb{Q}^{rc}((\mathbb{Q}))$.

Hint: Use without proof: $2^{\aleph_0} = \aleph_0^{\aleph_0}$.
A proof of this can be found for example in Chapter 12 of the book "Mengenlehre für den Mathematiker" by U. Friedrichsdorf and A. Prestel (Lemma 12.10 with $\alpha = \beta = 0$).

The exercise will be collected **Thursday, 18/06/2015** until 10.00 at box 13 near F 441.
