Universität Konstanz

Fachbereich
Mathematik und Statistik

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Real Algebraic Geometry II
Exercise Sheet 4

Note that due to the holiday on the 14/05/2015 the collection of this exercise has changed to Friday, 15/05/2015 until 10.00.

Exercise 1
Consider the vector space $(V, v) = (\mathbf{H}_{\gamma\in\mathbb{N}}\mathbb{R}, v_{\mathsf{min}})$.
Show that (V, v) does not admit a valuation basis.
Hint:
Show that $\left(\coprod_{\gamma\in\mathbb{N}}\mathbb{R}, v_{\mathsf{min}}\right) \subseteq (\mathbf{H}_{\gamma\in\mathbb{N}}\mathbb{R}, v_{\mathsf{min}})$ is a proper immediate extension.

Exercise 2
Let $S = \{a_\rho\}$ be a pseudo-convergent set. Show that

(a) if $v(a_\rho) < v(a_\sigma)$ for all $\rho < \sigma$, then $x = 0$ is a pseudo-limit of S.

(b) x is a pseudo-limit of S if and only if $v(x - a_\rho) < v(x - a_{\rho+1})$ for all ρ.

(c) if 0 is not a pseudo-limit of S and x is a pseudo-limit of S, then $v(x) = \mathsf{Ult}S$.

Exercise 3

(a) Consider $(V, v) = \left(\coprod_{\gamma \in \mathbb{N}_0} \mathbb{R}, v_{\text{min}}\right)$. For every $n \in \mathbb{N}_0$ define $a_n \in V$ by $a_n : \mathbb{N}_0 \longrightarrow \mathbb{R}$, $a_n(\gamma) := \begin{cases} 1 & , \text{ if } \gamma \leq n \\ 0 & , \text{ else} \end{cases}$, for all $\gamma \in \mathbb{N}_0$.

Show that the sequence $\{a_n \mid n \in \mathbb{N}_0\}$ does not have a pseudo-limit in (V, v).

(b) Let (V, v) be a Q-valued vector space with $v(V) = \mathbb{N}$. Let $S = \{a_\alpha \mid \alpha \in \Lambda\}$ be a pseudo-Cauchy sequence and x a pseudo-limit of S.

Show that x is a unique pseudo limit of S.

(c) Consider $(V, v) = \left(\coprod_{\gamma \in \mathbb{Q}} \mathbb{R}, v_{\text{min}}\right)$. For every $n \in \mathbb{N}_0$ define $a_n \in V$ by $a_n : \mathbb{Q} \longrightarrow \mathbb{R}$, $a_n(\gamma) := \begin{cases} 1 & , \text{ if } \gamma = \sum_{k=1}^{m} \frac{1}{k \cdot (k+1)} \text{ for some } m \in \{1, 2, \ldots, n\} \\ 0 & , \text{ else} \end{cases}$, for all $\gamma \in \mathbb{Q}$. Let $S = \{a_n \mid n \in \mathbb{N}_0\}$.

What is the breadth $B(S)$ of S?

The exercise will be collected **Friday, 15/05/2015 until 10.00** at box 13 near F 441.
