

Universität Konstanz

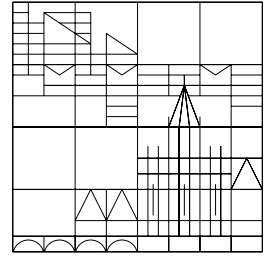
Fachbereich

Mathematik und Statistik

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## Real Algebraic Geometry II

### Exercise Sheet 4

Note that due to the holiday on the 14/05/2015 the collection of this exercise has changed to Friday, 15/05/2015 until 10.00.

#### Exercise 1

Consider the vector space  $(V, v) = (\mathbf{H}_{\gamma \in \mathbb{N}} \mathbb{R}, v_{\min})$ .

Show that  $(V, v)$  does not admit a valuation basis.

**Hint:**

Show that  $(\prod_{\gamma \in \mathbb{N}} \mathbb{R}, v_{\min}) \subseteq (\mathbf{H}_{\gamma \in \mathbb{N}} \mathbb{R}, v_{\min})$  is a proper immediate extension.

#### Exercise 2

Let  $S = \{a_\rho\}$  be a pseudo-convergent set. Show that

- if  $v(a_\rho) < v(a_\sigma)$  for all  $\rho < \sigma$ , then  $x = 0$  is a pseudo-limit of  $S$ .
- $x$  is a pseudo-limit of  $S$  if and only if  $v(x - a_\rho) < v(x - a_{\rho+1})$  for all  $\rho$ .
- if  $0$  is not a pseudo-limit of  $S$  and  $x$  is a pseudo-limit of  $S$ , then  $v(x) = \text{Ult}S$ .

### Exercise 3

- (a) Consider  $(V, v) = \left( \prod_{\gamma \in \mathbb{N}_0} \mathbb{R}, v_{\min} \right)$ . For every  $n \in \mathbb{N}_0$  define  $a_n \in V$  by
- $$a_n : \mathbb{N}_0 \longrightarrow \mathbb{R}, a_n(\gamma) := \begin{cases} 1 & , \text{ if } \gamma \leq n \\ 0 & , \text{ else} \end{cases}, \text{ for all } \gamma \in \mathbb{N}_0.$$

Show that the sequence  $\{a_n \mid n \in \mathbb{N}_0\}$  does not have a pseudo-limit in  $(V, v)$ .

- (b) Let  $(V, v)$  be a  $Q$ -valued vector space with  $v(V) = \mathbb{N}$ . Let  $S = \{a_\alpha \mid \alpha \in \Lambda\}$  be a pseudo-Cauchy sequence and  $x$  a pseudo-limit of  $S$ . Show that  $x$  is a unique pseudo limit of  $S$ .

- (c) Consider  $(V, v) = \left( \prod_{\gamma \in \mathbb{Q}} \mathbb{R}, v_{\min} \right)$ . For every  $n \in \mathbb{N}_0$  define  $a_n \in V$  by  $a_n : \mathbb{Q} \longrightarrow \mathbb{R}, a_n(\gamma) := \begin{cases} 1 & , \text{ if } \gamma = \sum_{k=1}^m \frac{1}{k \cdot (k+1)} \text{ for some } m \in \{1, 2, \dots, n\} \\ 0 & , \text{ else} \end{cases}$ , for all  $\gamma \in \mathbb{Q}$ . Let  $S = \{a_n \mid n \in \mathbb{N}_0\}$ .

What is the breadth  $B(S)$  of  $S$ ?

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The exercise will be collected **Friday, 15/05/2015 until 10.00** at box 13 near F 441.

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