Resolution of Singularities, Valuation Theory and Related Topics

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The automorphism group of a valued field

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Abstract

Let k be a field, G a totally ordered abelian group. The maximal field of generalised power series $k((G))$, endowed with its canonical valuation, plays a fundamental role in the classification of valued fields [2]. In the first part of this talk we report on [3], and describe the group $v - Aut(K)$ of valuation preserving automorphisms of a *Hahn field*, i.e. a subfield K of the maximal Hahn field, which contains the *minimal Hahn field* $k(G)$ (the fraction field of the group ring $k[G]$). Under the assumption that K is a *distinguished Hahn field*, i.e. satisfies a pair of lifting properties, we prove a structure theorem decomposing $v - Aut(K)$ into a 4-factor semi-direct product of notable subgroups. We identify a large class of distinguished Hahn fields [4]. We then focus on the group of *strongly additive automorphisms* of K (i.e. automorphisms commuting with infinite sums). We give an explicit description of the group of strongly additive internal automorphisms in terms of the groups $\mathrm{Hom}(G, k^{\times})$ (of homomorphisms of G into $k^{\times}$) and $\mathrm{Hom}(G, 1+M_v)$ (of homomorphisms of G into the group of 1-units of the valuation ring of K). To illustrate the power of our methods, we apply our results to some special cases, such as the field of Laurent series [6] and that of Puiseux series [1].

In the second part of this talk, we outline our work in progress [5] towards the aim of describing the automorphism group of a valued field (K, v). Under the assumption that K admits both a residue field and a value group section (and other conditions), a theorem of Kaplansky provides analytic embeddings ι of K such that $k(G) \subseteq \iota(K) \subseteq k((G))$ (where k is the residue field and G is the value group). To achieve our aim, it is necessary and sufficient to find conditions on (K, v), so that it admits a Kaplansky embedding ι in $k((G))$ for which $\iota(K)$ is a distinguished Hahn field. Finally, a further ongoing project is to understand how the automorphism group of (K, v) varies with v (in particular coarsenings, independent valuations, henselian valuations etc.).

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