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Salma Kuhlmann

Application of Jacobi's Representation Theorem to localy multiplicatively convex topological real algebras

Abstract:

Let A be a commutative unital real algebra and let ρ be a seminorm on A which satisfies $\rho(ab)\leq\rho(a)\rho(b)$. We apply T. Jacobi's representation theorem \cite{J} to determine the closure of a module S of A in the topology induced by ρ. We show that this closure is exactly the set of all elements $a\in A$ such that $\alpha(a)\ge0$ for every ρ-continuous real algebra homomorphism $\map{\alpha}{A}{\reals}$ with $\alpha(S)\subseteq[0,\infty)$, and that this result continues to hold when ρ is replaced by any locally multiplicatively convex topology τ on A. We obtain a representation of any linear functional $L : A \rightarrow \reals$ which is continuous with respect to any such ρ or τ and non-negative on S as integration with respect to a unique measure on the space of all real valued real-algebra homomorphisms on A, and we characterize the support of the measure obtained in this way.