Appendix to the Agreement on the Dual-Degree Program between Shanghai Jiaotong University (Department of Mathematics) and University of Konstanz (Department of Mathematics and Statistics)

Course Description for the Master Course in Mathematics

University of Konstanz – September 2007

Main Courses:

No.	Course Name	Credit	Description
1	Advanced Partial Differential Equations	9	This is a continuation of the basic course “Theory and Numerics of Partial Differential Equations”. Selected topics from the theory of partial differential equations are presented as there are: linear wave equations, spectral theory for general elliptic operators, variational evolution equations, conservation laws, hyperbolic systems – elasticity, Maxwell, coupled hyperbolic-parabolic systems, elliptic operators in unbounded domains, semigroup approach to evolution equations.
2	Advanced Numerics of Partial Differential Equations	9	The lecture is a continuation of the basic course “Theory and Numerics of Partial Differential Equations”. It focuses on numerical solution techniques for partial differential equations like finite volume, finite element and boundary element methods.
3	Algebraic Geometry	9	Continuation of the introduction to algebraic geometry, as begun in the basic course “Geometry”. Among the topics treated will be structural sheaves, rational maps and function fields, Zariski tangent spaces, dimension of varieties, regular and singular points, and on the algebraic side the Zariski spectrum, integral extensions and Noether normalization, and local rings.
4	Real Algebra	9	This course forms a unit together with the main cours “Real Algebraic Geometry”. Here the basics of real algebra are provided: Ordered fiels and real closures, real root counting, Tarsiki-Seidenberg, real spectrum, Stellensätze, the archimedean condition, representation theorems for positive polynomials.
5	Real Algebraic Geometry	9	This course forms a unit together with the main course “Real Algebra”. Here the emphasis is on the geometric side. Topics may include an introduction to the geometry of semialgebraic sets and the study of selected classes of real algebraci varieties, like curves or surfaces.
6	Mathematical Statistics	9	In this course a systematic introduction to the mathematical theory of statistical inference is given (estimation, testing, decision theory). Mathematical methods for stuydying properties of estimators, tests and related quantities are given, and the construction of statistical procedures is discussed in a general setup.
7	Time Series	9	A systematic introduction to time series analysis is given, with emphasis on understanding mathematical

	Analysis		foundations and their implications for data analysis. The spectral representation of stationary processes leads to an elegant theory in the Hilbert space of square integrable variables. Parametric and nonparametric statistical inference is discussed in the time and frequency domain. The practical application of the methods is illustrated by data examples.
8	Stochastic Processes I	9	Central limit theorem, basic notion of stochastic processes, semimartingales, Doob Meyer decomposition, Brownian motion, Itô calculus: stochastic integration, Itô formula, Tanaka formula, Feynman Kac formula, applications in control and finance.
9	Stochastic Processes II	9	Standard Black Scholes market, pricing in complete and incomplete markets, European and American options, arbitrage, free lunches, utility maximization, BSDE theory, Föllmer Schweizer hedging, Neyman Pearson hedging, optimal martingale measures and their convergence, exponential hedging, credit risk.

Special Courses (a selection each semester):

10	Nonlinear Evolution Equations	3	In this course methods to deal with nonlinear evolutions equations are presented. First a detailed study of wave equations demonstrates how to obtain solutions for small perturbations of equilibria (nonlinear Cauchy problem). Then a variety of further applications of the methods is given. Typical topics are perturbation theory, maximal regularity, quasilinear equations.
11	Dynamical Systems	3	The course gives an introduction into general approaches to deal with the asymptotics of nonlinear dynamical systems, which show a “chaotic” behavior despite an inherent dissipation. Typical examples will be the Lorenz attractor and Cahn-Hilliard systems.
12	Hyperbolic-Parabolic Coupled Systems	3	Various systems modeling hyperbolic and parabolic heat conduction coupled with elastic effects are considered. A theory of well-posedness and the large time asymptotic behavior of solutions is developed. Different systems are compared, first on an analytical level, but also for specific examples from the applications.
13	Parabolic Boundary Value Problems	3	In this course parabolic and parameter-elliptic boundary value problems and methods to solve them are discussed. Based on the Fourier- and the Laplace-transform, explicit representation formulas for the solutions are obtained. Topics include Shapiro-Lopatinskii condition, Michlin’s theorem, a priori-estimates and maximal regularity.
14	Pseudodifferential Operators	3	This course gives an introduction to the theory of pseudodifferential operators and their application to partial differential equations. Pseudodifferential operators are generalizations of differential operators which are defined in terms of the Fourier transform. Topics include composition and algebra property of pseudodifferential operators, symbol classes and norm estimates.
15	Numerics of Stochastic Differential Equations	3	In this course, numerical solution methods for stochastic differential equations are presented. The required generation of pseudo random numbers and the numerical evaluation of expected values is also discussed. Applications from physics and financial mathematics are given.

16	Asymptotic Analysis of Finite Difference Methods	3	The usual consistency analysis of finite difference methods can be related to regular asymptotic expansions of the numerical solutions with respect to the discretization parameter. In this course, additional tools from asymptotic analysis are introduced which allow to study various other properties of numerical schemes like initial, internal and boundary layers as well as long time behavior. As main example, lattice Boltzmann methods for the solution of transport equations like the incompressible Navier-Stokes equation are analyzed.
17	High-Dimensional Integration	3	High dimensional integrals appear, for example, in the form of expected values in probability theory but also in connection with solution methods for Fokker-Planck equations on high dimensional spaces as well as integro-differential equations like the Boltzmann equation. Due to the curse of dimensionality, usual quadrature rules cannot be used in high dimensional situations. Instead, Monte-Carlo, Quasi-Monte-Carlo, or sparse grid methods have to be employed which are presented in this lecture.
18	Numerical Methods in Fluid Dynamics	3	In this course, an introduction to numerical solution methods for flow problems (Stokes equations, Navier-Stokes equations, Euler equations) is given, including suitable finite difference, finite element and finite volume methods.
19	Quadratic Forms	3	Introduction to the algebraic theory of quadratic forms. Toipcs include the Witt ring of a field, Pfister's local-global principle, elementary theory of Pfister forms, reduced theory of quadratic forms.
20	Multivariate Statistics	3	In this course, an introduction to the analysis of multivariate data is given. This includes, in particulular, principal component analysis (PCA), factor analysis, discriminant analysis, cluster analysis, multidimensional scaling and canonical correlations.
21	Linear Models in Statistics	3	In this course, linear and related statistical models for quantitative and qualitative data are considered. For quantitative data, we start with linear regression and analysis of variance (ANOVA). General properties of estimates, tests and model checking are discussed. For qualitative or highly discreet data, generalized linear models such as logistic regression, Poisson regression etc. are introduced. Asymptotic theory as well as computational issues are discussed.
22	Design of Experiments and Sampling	3	Design of experiments deals with the question how to plan experiments in order to satisfy certain statistical optimality criteria. This leads, in particular, to questions in combinatorics and algebra. The theory of statistical sampling considers the question how to take a sample from a (usually finite) population in order to draw reliable statistical conclusions about the population. In this course, an introduction to both topics is given.
23	Bayesian Statistics	3	In this course, principles of Bayesian inference are introduced. Starting with iid normally distributed models and normal priors, the basic concepts are explained, and later generalized to non-iid and non-Gaussian models. Modern applicationos include in particular Markov Chain Monte Carlo, image processing and other spatial data, and time series.

Basic Courses (also for Bachelor students):

24	Number Theory	9	Introduction to algebraic number theory. Among the topics included will be algebraic integers, Dedekind domains, class groups, Minkowski theory, decomposition of primes in finite extensions, quadratic reciprocity, cylotomic fields.
25	Geometry	9	Introduction to the basic concepts of algebraic geometry: Polynomials, ideals, Hilbert Basissatz and Nullstellensatz, affine and projective varieties. Introduction to Groebner bases and basic algorithms for the operations on ideals. Use of a comuter algebra system.
26	Functional Analysis	9	The course is an introduction into the theory of mappings between general metric and normed spaces which is fundamental for further studies in Analysis and Numerics. Topics include: linear mappings in normed spaces, reflexivity, theorem of Baire and corollaries, compact mappings, Sobolev spaces, spectral theorem for self-adjoint operators.
27	Theory and Numerics of Partial Differential Equations	9	A survey on theoretical and applied aspects for partial differential equations (pde) is presented, focussing on the classification of the most important types and its discussion with analytical-theoretical and numerical methods as well as its relevance for applications. Programming and aquiring knowledge on program packages for finite elements is part of the course. Topics include: linear pde, elliptic pde (Perron's method), hyperbolic pde (separation of variables), parabolic pde (classical solutions, maximum principle), Hilbert space methods, numerical methods (finite differences, finite volumes, finite elements), consistency and stability.

Seminars (6 credits) are offered each semester in the various fields of Analysis and Numerics, Real Algebra and Geometry, and Stochastics