Universität Konstanz

Fachbereich
Mathematik und Statistik

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Real Algebraic Geometry II
Exercise Sheet 3

Exercise 1
Let (V, v) and (V_0, v_0) be valued vector spaces with $S(V) = S(V_0)$. Assume that $h : (V_0, v_0) \stackrel{\sim}{\longrightarrow} (V, v)$ is a value preserving isomorphism and $\mathcal{B}_0 \subseteq V_0 \setminus \{0\}$. Show that

(a) $\mathcal{B}_0$ is valuation independent if and only if $h(\mathcal{B}_0)$ is valuation independent.

(b) $\mathcal{B}_0$ is a valuation basis for (V_0, v_0) if and only if $h(\mathcal{B}_0)$ is a valuation basis for (V, v).

Exercise 2
Let (V, v) and (V_0, v_0) be Q-valued vector spaces with $S(V) = S(V_0)$. Let $\mathcal{B}_0 \subseteq V_0 \setminus \{0\}$ be a valuation basis for (V_0, v_0) and $h : \mathcal{B}_0 \longrightarrow \mathcal{B} \subseteq V$ be a value preserving bijection such that $\mathcal{B}$ is a valuation basis for (V, v). Extend h to a Q-vector space isomorphism $h : V_0 \longrightarrow V$ by linearity.
Show that h is valuation preserving.

Terminology:

- h is a value preserving bijection if $v(h(b)) = v_0(b)$ for all $b \in \mathcal{B}_0$.
- We extend h to a Q-vector space isomorphism $h : V_0 \longrightarrow V$ by linearity by setting $h(x) = h\left(\sum q_i b_i\right) := \sum q_i h(b_i)$ for all $x = \sum q_i b_i$.
- h is valuation preserving if $h : V_0 \longrightarrow V$ is a Q-valued vector space isomorphism.

Exercise 3

Consider the vector space $(V, v) = (\mathbf{H}_{\gamma\in\mathbb{N}} B_\gamma, v_{\min})$.

(a) Let $B_\gamma := \mathbb{Q}$ for all $\gamma \in \mathbb{N}$. Describe a maximal valuation independent set $\mathcal{B}$ such that support(b) is a singleton for every $b \in \mathcal{B}$.

(b) Let $B_\gamma := \mathbb{Q}$ for all $\gamma \in \mathbb{N}$. Describe a maximal valuation independent set $\mathcal{B}$ such that support(b) is infinite for every $b \in \mathcal{B}$.

(c) Let $B_\gamma := \mathbb{R}$ for all $\gamma \in \mathbb{N}$. Describe a maximal valuation independent set $\mathcal{B}$ such that support(b) is a singleton for every $b \in \mathcal{B}$.

(d) Let $B_\gamma := \mathbb{R}$ for all $\gamma \in \mathbb{N}$. Describe a maximal valuation independent set $\mathcal{B}$ such that support(b) is infinite for every $b \in \mathcal{B}$.

(e) Let $B_\gamma := \mathbb{Q}$ for all $\gamma \in \mathbb{N}$ even and let $B_\gamma := \mathbb{R}$ for all $\gamma \in \mathbb{N}$ odd. Describe a maximal valuation independent set $\mathcal{B}$ such that support(b) is infinite for every $b \in \mathcal{B}$.

The exercise will be collected **Thursday, 07/05/2015** until 10.00 at box 13 near F 441.
