

Universität Konstanz

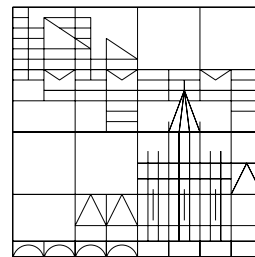
Fachbereich

Mathematik und Statistik

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## Real Algebraic Geometry II

### Exercise Sheet 3

#### Exercise 1

Let  $(V, v)$  and  $(V_0, v_0)$  be valued vector spaces with  $S(V) = S(V_0)$ . Assume that  $h : (V_0, v_0) \xrightarrow{\sim} (V, v)$  is a value preserving isomorphism and  $\mathcal{B}_0 \subseteq V_0 \setminus \{0\}$ . Show that

- $\mathcal{B}_0$  is valuation independent if and only if  $h(\mathcal{B}_0)$  is valuation independent.
- $\mathcal{B}_0$  is a valuation basis for  $(V_0, v_0)$  if and only if  $h(\mathcal{B}_0)$  is a valuation basis for  $(V, v)$ .

#### Exercise 2

Let  $(V, v)$  and  $(V_0, v_0)$  be  $Q$ -valued vector spaces with  $S(V) = S(V_0)$ . Let  $\mathcal{B}_0 \subseteq V_0 \setminus \{0\}$  be a valuation basis for  $(V_0, v_0)$  and  $h : \mathcal{B}_0 \rightarrow \mathcal{B} \subseteq V$  be a value preserving bijection such that  $\mathcal{B}$  is a valuation basis for  $(V, v)$ . Extend  $h$  to a  $Q$ -vector space isomorphism  $h : V_0 \rightarrow V$  by linearity. Show that  $h$  is valuation preserving.

#### Terminology:

- $h$  is a value preserving bijection if  $v(h(b)) = v_0(b)$  for all  $b \in \mathcal{B}_0$ .
- We extend  $h$  to a  $Q$ -vector space isomorphism  $h : V_0 \rightarrow V$  by linearity by setting  $h(x) = h(\sum q_i b_i) := \sum q_i h(b_i)$  for all  $x = \sum q_i b_i$ .
- $h$  is valuation preserving if  $h : V_0 \rightarrow V$  is a  $Q$ -valued vector space isomorphism.

### Exercise 3

Consider the vector space  $(V, v) = (\mathbf{H}_{\gamma \in \mathbb{N}} B_\gamma, v_{\min})$ .

- (a) Let  $B_\gamma := \mathbb{Q}$  for all  $\gamma \in \mathbb{N}$ . Describe a maximal valuation independent set  $\mathcal{B}$  such that  $\text{support}(b)$  is a singleton for every  $b \in \mathcal{B}$ .
- (b) Let  $B_\gamma := \mathbb{Q}$  for all  $\gamma \in \mathbb{N}$ . Describe a maximal valuation independent set  $\mathcal{B}$  such that  $\text{support}(b)$  is infinite for every  $b \in \mathcal{B}$ .
- (c) Let  $B_\gamma := \mathbb{R}$  for all  $\gamma \in \mathbb{N}$ . Describe a maximal valuation independent set  $\mathcal{B}$  such that  $\text{support}(b)$  is a singleton for every  $b \in \mathcal{B}$ .
- (d) Let  $B_\gamma := \mathbb{R}$  for all  $\gamma \in \mathbb{N}$ . Describe a maximal valuation independent set  $\mathcal{B}$  such that  $\text{support}(b)$  is infinite for every  $b \in \mathcal{B}$ .
- (e) Let  $B_\gamma := \mathbb{Q}$  for all  $\gamma \in \mathbb{N}$  even and let  $B_\gamma := \mathbb{R}$  for all  $\gamma \in \mathbb{N}$  odd. Describe a maximal valuation independent set  $\mathcal{B}$  such that  $\text{support}(b)$  is infinite for every  $b \in \mathcal{B}$ .

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The exercise will be collected **Thursday, 07/05/2015** until 10.00 at box 13 near F 441.

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<http://www.math.uni-konstanz.de/~dupont/rag.htm>