MODEL THEORY OF INFINITARY LOGIC WITH ALGEBRAIC APPLICATIONS

SEMINAR im WS2010-2011

"Infinitäre Logik mit algebraischen Anwendungen"

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- Vorbesprechung: 12. Oktober 2010 um 14.00 Uhr, F433
- Termin: Dienstags 14 bis 16 Uhr oder n.V.
- Beginn: Dienstag, 19. Oktober 2010

In diesem Seminar sollen die Grundlagen der Infinitärer Logik erarbeitet werden. Dabei ist der Zugang zum Thema algebraisch. Wir werden wichtige Anwendungen in der Gruppentheorie studieren. Grundlage bildet das Kapitel: "Back and Forth through infinitary logics" von J. Barwise, in Studies in Model Theory, Band 8 im MAA, 1973. Die Teilnehmerinnen und Teilnehmer werden bei der Ausarbeitung ihrer Vorträge durch individuelle Vorbesprechungen unterstützt.

Voraussetzungen: Das Seminar richtet sich in erster Linie an Studierende des Hauptstudiums (5. Semester), ist aber auch für höhere Semester geeignet. Vorkenntnisse in der Logik oder Modelltheorie sind hilfreich, aber nicht erforderlich.

Zielgruppe: LA, BA, D, MA

Week by week progress

Week 1 – Introduction (given by Itay). Reminder of first order logic: formulas, sentences, structures, substructures, isomorphisms, homomorphisms, truth in a structure, compactness and completeness theorem.
Introduction to ordinals and cardinals.

Week 2 – Back-and-Forth. (Section 1 of [1])
Cantor's theorem: Any countable dense linear ordered set without endpoints is isomorphic to $(\mathbb{Q}, <)$.
Definition of the "Back-and-Forth" property and of the relation $\cong_p$ between structures (saying that two structures have a family of partial isomorphisms between them).
Equivalent definition in terms of games.
Some examples: Atomless Boolean algebras, Thin abelian groups (may be postponed to the week after).

Week 3 – The language $L_{\infty,\omega}$. (Section 2 of [1]).
Definition of infinitary languages $L_{\kappa,\lambda}$. Examples of statements that cannot be expressed in first order but can be expressed in infinitary languages.
Definition of an elementary substructure, and a version of Lowenheim-Skolem theorem.
Definition of $\equiv_{\infty,\omega}$ (having the same true sentences in $L_{\infty,\omega}$).

Week 4 – Karp's Theorem. (Section 3 of [1])
Proof of Karp's theorem that states that $\cong_p = \equiv_{\infty,\omega}$.
A few applications (products are well defined, direct sums are well defined, and perhaps we shall have a look in the first order case as well).

Week 5 – More applications and variants of Karp's theorem. (Section 3 of [1] – cont.).
2 strengthening of Karp's theorem, and application to direct sums.

Week 6 – Scott's theorem. (Section 4 of [1])
Scott's theorem states that for ant countable language L and a countable structure M, there is a sentence φ in $L_{\omega_1,\omega}$ such that $N \models \varphi$ iff $M \cong N$.
This is a generalization of a similar result in the first order case.
Proof of the theorem.
Are the countable demands needed?

Week 7 – Applications of Scott's theorem. (Section 4 of [1] cont.)
Showing that a version of the Continuum Hypothesis is true for the number of automorphisms of a countable structure.

Week 8 – $\aleph_1$-free Abelian groups. (Section 5 of [1])
An Abelian group is $\aleph_1$-free if all countable subgroups are free.
Show the following theorem: G is $\aleph_1$-free iff it is $\equiv_{\infty,\omega}$ to the free group on $\aleph_0$ generators.
Conclude that being free is not expressible in $L_{\infty,\omega}$, while being $\aleph_1$-free is.

Week 9 – Generalization of Ulm's Theorem. (Section 6 of [1])
Ulm's theorem gives a set of invariants that determine a reduced countable p-group (a group all of whose elements have order a power of p).
The generalization is that in fact, even if the groups are not countable, these invariants determine the $\equiv_{\infty,\omega}$ class of the groups.
This is a very nice theorem, but the proof is a bit involved so there will be two (or more) talks about it.
The proof uses ideas from the proof of Ulm's theorem in [2].

Week 10 – Cont. of Week 9. (Section 6 of [1])

Week 11 – Return to Scott's theorem. Finish the proof of the theorem (if not finished).
Prove the theorem for uncountable languages (the proof can be found in [4, page 57]).

Week 12 – Cont. of Week 11 and Completeness. Finish the proof of Scott's theorem for uncountable languages (if not finished).
Looking at the completeness theorem for $L_{\omega_1,\omega}$ (Chapters 3 and 4 in [3]): Finding a set of axioms and deductions that gives a completeness theorem for $L_{\omega_1,\omega}$.

Week 13 – Completeness cont. (Chapters 3 and 4 in [3]).

Week 14 – Summary (given by Itay). Summary of the seminar and a look to the future.

References

[1] Jon Barwise. Back and forth through infinitary logics. In *Studies in Model Theory*, volume 8 of *MAA*, pages 5–34. 1973.
[2] I Kaplansky. *Infinite Abelian groups*. Ann Arbor: University of Michigan Press, 1968.
[3] H. J. Keisler. *Model theory for infinitary logic*. North-Holland Publishing Co., Amsterdam, 1971.
[4] David Marker. *Model Theory: An Introduction*. Springer, 2002.