GOETHE UNIVERSITÄT FRANKFURT AM MAIN

Frankfurter Seminar

Kolloquium des Instituts für Mathematik

Wintersemester 2023/24

Frankfurter Seminar, 12. Dezember 2023

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Positive Polynomials and Moment Problems

Hilbert's 17th problem asked whether a real polynomial $p(x_1, \cdots, x_n)$ which takes nonnegative values as a function on $\mathbb{R}^n$ is a finite sum of squares (SOS) of real rational functions $q(x_1, \cdots, x_n)/r(x_1, \cdots, x_n)$. A complete positive answer was provided by Artin and Schreier, giving birth to real algebraic geometry. The question when the SOS representation is denominator free, i.e. can be achieved using SOS of polynomials rather than of rational functions, remained however of particular interest. In his pioneering 1888 paper, Hilbert gave a general answer (in terms of the degree of the polynomial and its number of variables). Subsequent general results, such as the Positivstellensatz, pertain to a relative situation, where one considers polynomials non-negative on a basic closed semialgebraic subset K of $\mathbb{R}^n$ and seeks SOS representations, albeit weighted by the finitely many polynomial inequalities defining K. Stronger results hold when K is compact; the Archimedean Positivstellensatz became a fundamental tool in theory and applications.

By the classical Riesz-Haviland theorem, the problem of characterizing positive polynomials on a given closed subset K of $\mathbb{R}^n$ is dual to the finite dimensional moment problem (i.e. that of representing a linear functional on the polynomial algebra $\mathbb{R}[x_1, \cdots, x_n]$ as integration with respect to a Borel measure). A more recent algebraic approach was promoted in a series of papers (that I myself co-authored) which study the moment problem on a general not necessarily finitely generated commutative unital real algebra (instead of the finitely generated polynomial algebra), a context adapted to infinite dimensional moment problems.

In this talk I will survey (with examples) various Positivstellensätze and their corresponding moment problem interpretations.

Tee ab 16:15 Uhr

Robert-Mayer-Straße 10 | Raum 711

Ginkgo-Seminar 15:15 - 16:00 Uhr

Constantin Ickstadt Sums of squares and convex optimization

Teilnahme nur für Studierende, Promovierende und Postdocs

Tee 16:15 - 16:45 Uhr

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