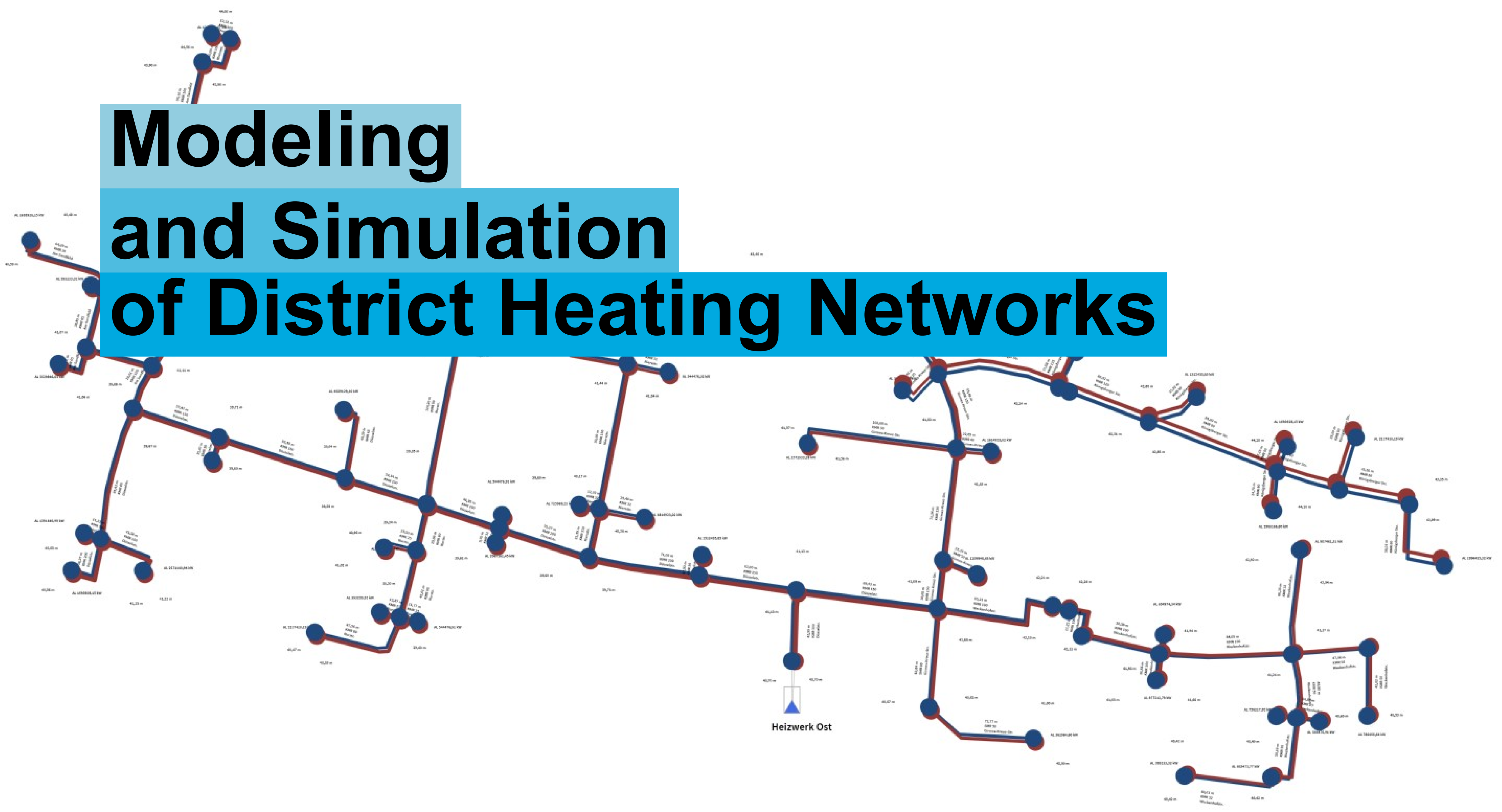
Modeling and Simulation of District Heating Networks



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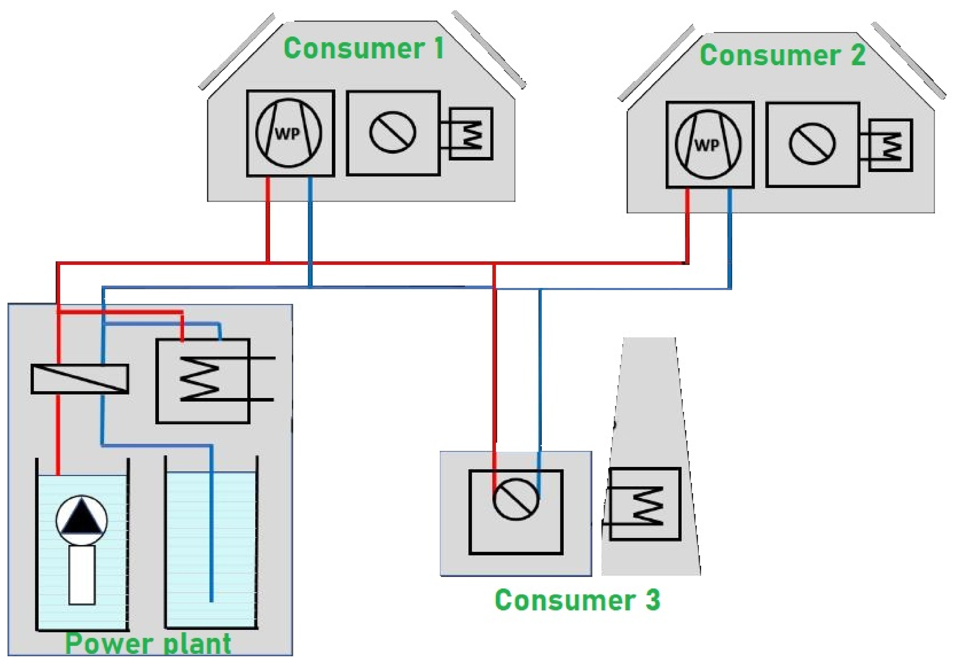
Aim of the project

- Mathematical modeling of district heating networks with decentralized feed-in as **differential algebraic equations** (DAEs)
- Certified and efficient dynamic simulation of DAE models
- Optimization and control of district heating networks



Physical background

To model the transport in the pipes we use a simplified form of the incompressible **Navier-Stokes-equations**

$$\partial_t T(t,x) + v(t)\partial_x T(t,x) = -\frac{4k}{c_p d\rho}(T(t,x) - T_{ext}) \qquad \text{(energy eq.)}$$

$$\rho\partial_t v(t) + \frac{\Delta p(t)}{L} = -\frac{\lambda}{2d}|v(t)|v(t)\rho - g(\partial_x h)\rho \qquad \text{(impuls eq.)}$$

Coupling conditions in every node j

$$\sum_{i\in\sigma_j^{in}} A_i\rho v_i = \sum_{i\in\sigma_j^{out}} A_i\rho v_i \qquad \text{(conserv. mass)}$$

$$\sum_{i\in\sigma_j^{in}} c_p A_i\rho v_i(t) T_i(t,L_i) = \sum_{i\in\sigma_j^{out}} c_p A_i\rho v_i(t) T_i(t,0) \qquad \text{(conserv. energy)}$$

$$p_i(t,L_i) = p_l(t,0) \text{ for all } i\in\sigma_j^{in}, l\in\sigma_j^{out} \qquad \text{(continuity)}$$

$$T_l(t,0) = T_l(t,0) \text{ for all } i,l\in\sigma_j^{out} \qquad \text{(perfect mixing)}$$

Consumer equations

$$v_{in}(t) = v_{out}(t)$$

$$Q_k(t) = c_p A\rho v_{in}(T_{in}(t,L_{in}) - T_{out}(t,0))$$

Together with some graph theoretical modeling we formulate the system as a **semi-explicit differential algebraic equation** (DAE).

$$\begin{aligned} \dot{x}_1(t) &= f_1(t,x_1(t),x_2(t),y(t)), & 0 &= g_1(t,x_1(t),x_2(t),y(t)) \\ \dot{x}_2(t) &= f_2(t,x_1(t),x_2(t),y(t)), & 0 &= g_2(t,x_1(t),x_2(t)) \end{aligned} \qquad (1)$$

Theoretical results

Neglecting the term $\partial_t v$ in the impulse equation leads to an index 1 DAE. Otherwise the system is index 2. Define the **set of all consistent initial values** for given $t\in I$

$$\mathcal{M}_0(t) := \{(x,y)\in\mathcal{D}_x\times\mathcal{D}_y \,|\, g(t,x,y)=0\}$$

for the index 1 case and also the set for the index 2 case

$$\mathcal{M}_1(t) := \{(x,y)\in\mathcal{D}_x\times\mathcal{D}_y \,|\, g(t,x,y)=0,\ \exists w\in\mathbb{R}^{n_y} \text{ with } \partial_t g(t,x,y) + \partial_x g(t,x,y) f(t,x,y) + \partial_y g(t,x,y) w = 0\}$$

we get the following existing results.

Theorem 1 *Let the semi-explicit DAE* (1) *have differentiation index* $di = 1$ *on* $I\times\mathcal{D}_x\times\mathcal{D}_y$. *Then for* $t_0\in I$ *and* $(x_0,y_0)\in\mathcal{M}_0(t_0)\cap(\mathcal{D}_x\times\mathcal{D}_y)$ *there exists a locally unique solution* $x:\tilde{I}\to\mathbb{R}^{n_x}$, $y:\tilde{I}\to\mathbb{R}^{n_y}$ *in* C^1 *with* $t_0\in\tilde{I}\subset I$ *an open interval and* $x(t_0)=x_0$ *and* $y(t_0)=y_0$.

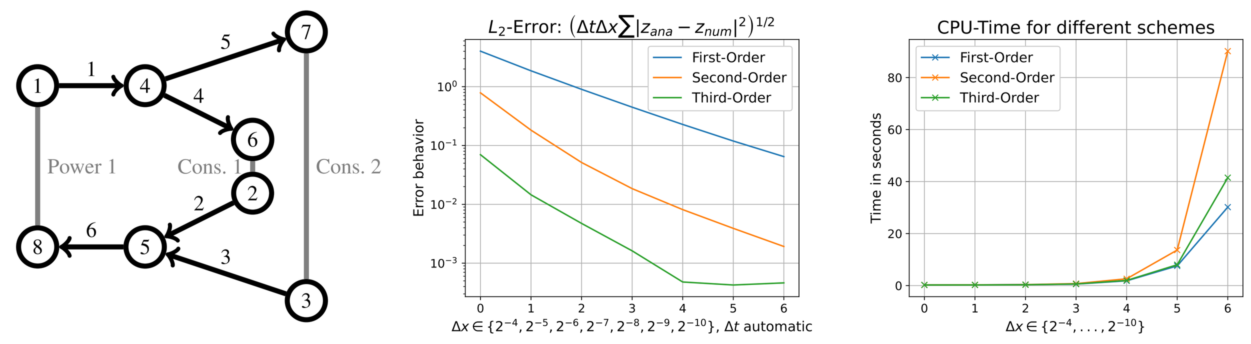
Theorem 2 *Suppose that the semi-explicit DAE* (1) *has differentiation index* $di = 2$ *on* $I\times\mathcal{D}_x\times\mathcal{D}_y$. *Moreover, it holds*

$$\ker\partial_y g(t,x,y) \text{ does not depend on } (t,x,y)\in I\times\mathcal{D}_x\times\mathcal{D}_y.$$

Then for $x_0,y_0\in\mathcal{M}_1(t_0)\cap\mathcal{D}_x\times\mathcal{D}_y$ *with* $t_0\in I$ *there exists a locally unique solution* $x:\tilde{I}\to\mathcal{D}_x$, $y:\tilde{I}\to\mathcal{D}_y$ *in* C^1 *with* $x(t_0)=x_0$ *and* $y(t_0)=y_0$.

Different space discretization

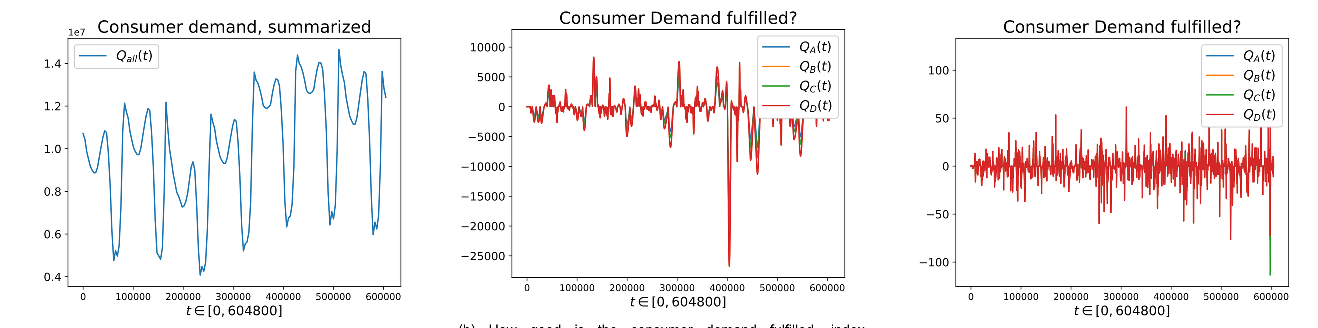
We compare different spatial discretizations for both index problems, namely a first-, second and third-order discretization. The expected order is clearly visible in all three cases.



Neither the index 1 nor the index 2 case is significantly better than the other. In any case it is not worthwhile to simulate with the order 2 method, since the third order method is both more exact and faster in the sense of CPU time.

Simulation results

Comparing in the solution of the numerical simulation how good the consumer demand is fulfilled for one week where the total consumption of all consumers are given as:



(a) Total consumption of all consumers for one week. (b) How good is the consumer demand fulfilled, index 1. (c) How good is the consumer demand fulfilled, index 2.

Comparing the solution of the index 2 system with the solution of the index 1 system in the Euclidean norm we get as difference

$$\sqrt{\sum_{k=1}^{n_t}\Delta t\|z_{full}^k - z_{red}^k\|_2^2} = 0.0908, \qquad \text{with } \Delta t \text{ maximum stepsize.}$$

We can also compare the difference of the velocities and pressures at certain consumers A, B, C and D in the relative L_2–norm.

Discrete relative L_2–norm:

	A	B	C	D
Velocity	1.6e-3	1.6e-3	1.6e-3	1.6e-3
Pressure	4.5e-4	7.2e-4	8.7e-4	3.4e-4

Outlook

- Develop efficient nonlinear optimization methods to get optimal solutions for the network.

$$\min \int_{t_0}^{t_f}\sum_{i=1}^{3}\omega_i(t)u_i(t)dt + \frac{\varepsilon}{2}\left(\|x-x_d\|_{L_2}^2 + \|y-y_d\|_{L_2}^2 + \|u\|_{L_2}^2\right)$$

- Provide port-Hamiltonian formulations to apply control theoretic strategies.
- Applying model predictive control for stabilizing the systems of first-order quasilinear hyperbolic equations.

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