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Separating Cones for the SOS and PSD Cones that Fail to be Spectrahedral Shadows

A Talk by Sarah Hess, University of Konstanz, in Real Geometry and Algebra Seminar and „KWIM“ Lecture Series

Friday, 15^{th} December 2023, 13:30 PM in F426

Abstract: For $n, d \in \mathbb{N}$, let $\mathcal{P}_{n+1,2d}$ denote the cone of positive semi-definite (PSD) homogeneous polynomials (forms) in $n+1$ variables of degree $2d$ with real coefficients. $\mathcal{P}_{n+1,2d}$ contains the subcone $\Sigma_{n+1,2d}$ of forms that are representable as finite sums of squares (SOS) of forms. By Hilbert's 1888 Theorem, $\Sigma_{n+1,2d} = \mathcal{P}_{n+1,2d}$ exactly in the *Hilbert cases* $(n+1, 2d)$ with $n+1 = 2$ or $2d = 2$ or $(3, 4)$.

In this talk, we introduce, for the *non-Hilbert cases*, a specific cone filtration

$$\Sigma_{n+1,2d} = C_0 \subseteq C_1 \subseteq \ldots \subseteq C_{k(n,d)-n-1} \subseteq C_{k(n,d)-n} = \mathcal{P}_{n+1,2d}, \tag{0.1}$$

defined via the Gram matrix method. We determine the number $\mu(n,d)$ of strictly separating intermediate cones (i.e., C_i such that $\Sigma_{n+1,2d} \subsetneq C_i \subsetneq \mathcal{P}_{n+1,2d}$) in (0.1) and, thus, reduce (0.1) to a specific cone subfiltration

$$\Sigma_{n+1,2d} = C'_0 \subsetneq C'_1 \subsetneq \ldots \subsetneq C'_{\mu(n,d)} \subsetneq C'_{\mu(n,d)+1} = \mathcal{P}_{n+1,2d}. \tag{0.2}$$

For $i = 1, \ldots, \mu(n,d)$, we show that each C'_i in (0.2) fails to be a spectrahedral shadow by constructing explicit examples of separating forms $f \in C'_i \backslash C'_{i-1}$ and inductively applying the methods of Scheiderer from 2018.

This is a joint work with Charu Goel and Salma Kuhlmann.

Remark: Directly after the talk there will be a KWIM Tea and Coffee Round.

Real Geometry and Algebra Group

Konstanz Women in Mathematics (KWIM)