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ON GRADIENT METHODS FOR INTERFERENCE CANCELATION

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ABSTRACT. In this note we discuss the pre-filtering unit as a part of the receiver in GSM mobile stations. Different cost functions lead to different filter coefficients. We discuss some approaches and compare their performance under the presence of a co-channel interferer. In particular, we consider a version of a constant modulus algorithm (CMA) and gradient search connected with CMA.

1. Introduction

Due to the cell structure of mobile communication systems like GSM and EDGE, the presence of unwanted interferers at the same time and frequency can – in principle – be avoided. However, when frequency hopping is applied there is some probability that at least one mobile station in the neighboring cell uses the same frequency and a time slot which has non-empty overlap with the user's time slot. Such type of interference is called co-channel interference in contrast to the adjacent channel interference where the interfering signal uses another frequency.

During the last years, there has been an increasing demand on mobile communication receivers for handling adjacent and co-channel interference to increase frequency load and network capacity. In fact, single antenna interference cancelation (SAIC), also called ARP (advanced receiver performance), will become mandatory in future releases of the GERAN GSM standard [1]. Thus, there have been many recent approaches to address this problem.

In contrast to the user signal, no known pilot symbols are available for the unknown interferer. Basically, we are in the situation of blind estimation of the interferer signal. There are different approaches to interferer cancelation. The present paper concentrates on cancelation by modifying the criteria for the equalization of the user signal. Other methods include, for instance, blind estimation of the interferer signal followed by subtraction of the estimated interferer signal from the received signal.

To explain the modification for the interferer case, we will shortly discuss a standard receiver structure. Usually, a GSM mobile station receiver performs some pre-filtering of the received symbols in order to improve the subsequent data detection (in most cases done by a Viterbi type algorithm). The coefficients of the FIR or IIR pre-filter are chosen in accordance with an optimality criterion. In particular, if $z(k)$ denotes the symbol number k after the pre-filtering and if $s(k)$ stands for the correct (transmitted) symbol, the square error $|z(k) - s(k)|^2$ may be used as a cost function. An optimal pre-filtering structure in this sense is one which minimizes the mean square error $\mathrm{E}[|z(k) - s(k)|^2]$. We will also speak of the MSE approach. As a general reference for receiver structures and optimality of filters in this context, we mention [2].

The pre-filter coefficients in the MSE approach can easily be computed if an estimate for the channel impulse response is known. This is the case in most mobile communication receivers. Moreover, there exist efficient implementations for the computation of the pre-filter coefficients (see [3]).

In the presence of interferers, the sketched approach has two main disadvantages:
- The optimality of the coefficients in the MSE sense does only hold in the case of white noise (and the interferer destroys the whiteness of the noise).
- The number of known training symbols (midamble symbols) is too small for sufficient performance in an interferer scenario.

We will discuss the performance of the standard MSE approach and variants thereof in the subsequent sections. We will see that it is advantageous not to use explicit channel estimates, and we will discuss a modified cost function which can include the whole burst instead of the midamble only. This cost function was defined by Kuzminskiy [4] and is based on the Constant Modulus Algorithm (CMA) known from the theory of blind detection. The constant modulus cost function is defined without knowledge of the correct symbol and can be applied to unknown user data, thus enlarging the number of symbols which can be used for pre-filter coefficients computation.

The pre-filter coefficients minimizing Kuzminskiy's cost function are solutions of a non-linear equation system and, consequently, can be computed only iteratively. Here, the iteration is usually based on some variant of gradient search methods. The aim of the present note is to give some performance results for the different methods mentioned above.

The paper is organized as follows. In Section 2, we will introduce our signal model and the pre-filtering structure we are considering. In Section 3, we will compare the MSE approach with and without use of explicit channel estimates. In Section 4, the CMA approach will be discussed, and in the last section conclusions can be found.

2. Signal model and receiver structures

Equalization in GSM systems is based on the training sequence which is transmitted as the midamble of every burst. Here a known sequence of pilot symbols is transmitted which is the basis for channel estimation and subsequent data detection. Whereas in GSM the Gaussian Minimum Shift Keying (GMSK) modulation is used, for EDGE systems 8PSK modulation is applied. For the present paper we restrict ourselves to the GSM case and approximate GMSK modulation, as usual, by BPSK. An idealized model of the received signal is given by

$$x(k) = \sum_{\ell=0}^{L} h_\ell s(k-\ell) + n(k) \tag{1}$$

where $(s(k))_{k=0,\ldots,N_b}$ is the sequence of the transmitted signal over the whole burst (including midamble), $h = (h_0, \ldots, h_L)^T$ denotes the channel impulse response which is assumed to be of order L, and $(n(k))_{k=1,\ldots,N_b+L}$ stands for additional white noise.

Note that in the signal model above we ignored non-ideal effects like additional DC, IQ imbalance and other non-linear distortions. Moreover, the additional symbol-by-symbol $\frac{\pi}{2}$-rotation in GSM systems is not included in the signal model as we assume corresponding de-rotation in the receiver. We will also consider the channel impulse response to be approximately constant within the duration of one burst. We remark that the latter assumption does not hold for large velocities of the mobile station, but in a first step SAIC will be introduced for low velocities like in the GERAN model where velocity is restricted to 3 km/h.

In the presence of co-channel interference, it is reasonable to refine the signal model (1). In addition to the user signal there is one or more interferer signals contained in the received signal. This can be modeled explicitly, or we can include the interferer(s) into the noise component $n(k)$ resulting in non-white noise.

As already mentioned in the introduction, a typical receiver structure includes a pre-filtering unit. We will distinguish between the feed-forward part (FIR filter of order N) and the feedback part (IIR filter of order L_g) as shown in Figure 1. The feedback part may be present or not.

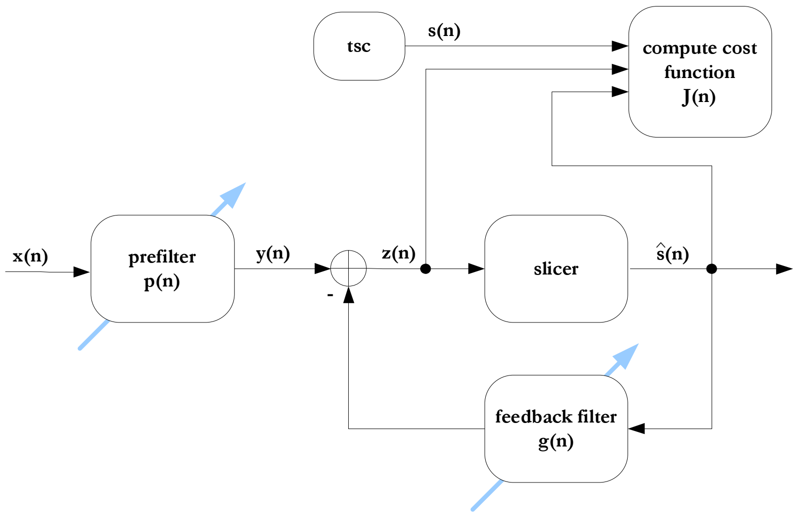

FIGURE 1. General receiver structure.

We introduce some notation. For $k = 0, 1, \ldots$ we set $\mathbf{x}(k) := (x(k), x(k+1), \ldots, x(k+N))^T$. The feedforward filter coefficients are denoted by $\mathbf{p} = (p_0, \ldots, p_N)^T$, and the feedback filter coefficients by $\mathbf{g} = (g_1, \dots, g_L)^T$. The signal after feedforward filtering will be denoted by $y(k)$, the signal after feedback filtering by $z(k)$. The sequence $z(k)$ is the input of the slicer, the decisions are denoted by $\hat{s}(k)$. We assume the absence of decision errors, so we have $\hat{s}(k) = s(k)$. The part of decided symbols used for the feedback filter will be denoted by $\mathbf{s}_F(k) = (s(k-1), \dots, s(k-L))^T$. With these notations, the signal after feedforward filtering is given as

$$y(k) = \mathbf{p}^H \cdot \mathbf{x}(k), \tag{2}$$

where $(\cdot)^H$ denotes the complex conjugate transposed vector. The signal after feedback filtering equals

$$z(k) = y(k) - \mathbf{g}^H \cdot \mathbf{s}_F(k) = \mathbf{p}^H \cdot \mathbf{x}(k) - \mathbf{g}^H \cdot \mathbf{s}_F(k). \tag{3}$$

3. MSE and LMS approach

To describe the MSE approach, we start with white noise in the signal model (1) with variance $\mathrm{E}[|n(k)|^2] = \sigma_{\mathbf{n}}^2$, and with a known channel impulse response. The user data $s(k)$ are also assumed to be white with variance $\sigma_{\mathbf{s}}^2$ and independent of the additive noise $n(k)$. Note that the assumption of white user data is reasonable due to channel coding which eliminates, to some extent, asymmetries in the user data. Under these assumptions, the optimal (in the sense of minimal mean square error) prefilter coefficients $\mathbf{p}_{\mathrm{MSE}}$ can be computed explicitly. In the interferer scenario optimality is no longer given. The MSE approach consists in using $\mathbf{p}_{\mathrm{MSE}}$ also in the interferer case. As an estimate for the channel impulse response h enters the formula for $\mathbf{p}_{\mathrm{MSE}}$, we will also speak of the h-based method.

We define the channel matrix $\mathbf{H} \in \mathbb{C}^{(N+1)\times(N+L+1)}$ by

$$\mathbf{H} := \begin{pmatrix} h_0 & h_1 & \cdots & h_L & 0 & \cdots & 0 \\ 0 & h_0 & h_1 & \cdots & h_L & \ddots & \vdots \\ \vdots & & \ddots & \ddots & & \ddots & 0 \\ 0 & \cdots & 0 & h_0 & h_1 & \cdots & h_L \end{pmatrix}.$$

We split $\mathbf{H}$ into two blocks

$$\mathbf{H} = \left[\ \mathbf{H}_1 \ \middle|\ \mathbf{H}_2\ \right]$$

with $\mathbf{H}_1 \in \mathbb{C}^{(N+1)\times(N+1)}$ and $\mathbf{H}_2 \in \mathbb{C}^{(N+1)\times L}$ and set $\mathbf{e}_{N+1} := (0, \dots, 0, 1)^T \in \mathbb{C}^{N+1}$. Then the MSE feedforward coefficients are given by (see, e.g., [3])

$$\mathbf{p}_{\mathrm{MSE}}^{(h)} = \left(\mathbf{H}_1\mathbf{H}_1^H + \frac{\sigma_{\mathbf{n}}^2}{\sigma_{\mathbf{s}}^2}\mathbf{I}\right)^{-1}\mathbf{H}_1\mathbf{e}_{N+1}, \tag{4}$$

and the MSE feedback coefficients are given by

$$\mathbf{g}_{\mathrm{MSE}}^{(h)} = \mathbf{H}_2\mathbf{p}_{\mathrm{MSE}}^{(h)}. \tag{5}$$

Now let us consider the interferer case. Here the noise is no longer white, and a least mean square (LMS) approach appears to be more suitable. Here we minimize

$$J_{\text{LMS}}(\mathbf{p}) := \frac{1}{N_0} \sum_{n=n_0+1}^{n_0+N_0} |e(n)|^2 \tag{6}$$

with $|e(n)|^2 = |z(n) - s(n)|^2$, where the sum is taken over the midamble symbols which are assumed to have indices $n_0+1, \dots, n_0+N_0$. We define the received sample matrix

$$\mathbf{A}_x := \big(\mathbf{x}(n_0+1), \cdots, \mathbf{x}(n_0+N_0)\big)^H \in \mathbb{C}^{N_0 \times (N+1)}$$

and the data symbol matrix

$$\mathbf{A}_s := \big(\mathbf{s}_F(n_0), \cdots, \mathbf{s}_F(n_0+N_0)\big)^H \in \mathbb{C}^{N_0 \times L}.$$

We further set $\mathbf{A}_{xs} := [\mathbf{A}_x \; \mathbf{A}_s] \in \mathbb{C}^{N_0 \times (N+L+1)}$ and $\mathbf{s} := (s(n_0+1), \dots, s(n_0+N_0))^T$. We can then easily compute the LMS filter coefficients which equal

$$\begin{pmatrix} \mathbf{p}_{\text{LMS}} \\ -\mathbf{g}_{\text{LMS}} \end{pmatrix} = \big(\mathbf{A}_{xs}^H \mathbf{A}_{xs}\big)^{-1} \mathbf{A}_{xs}^H \mathbf{s}^*. \tag{7}$$

Here $(\quad)^*$ denotes complex conjugation.

Let us remark that the LMS approach can be interpreted as a stochastic approach with the mean value $\mathrm{E}[x(n)x(n)^H]$ being replaced by the averaged sample correlation

$$\frac{1}{N_0} \sum_{n=n_0+1}^{n_0+N_0} x(n)x(n)^H.$$

In the LMS method we don't use estimates of the channel h but directly the input samples x. Thus, we will also speak of x-based filtering.

Figure 2 shows the comparison of the two approaches. The bit error rate (BER) is shown as a function of the carrier-to-interferer ratio (CIR) with signal-to-noise ratio (SNR) being fixed to 20 dB. All simulation results presented here are based on 5000 GSM frames. We used the GERAN SAIC model defined in [1]. As expected, it can be seen in Figure 2 that x-based prefiltering shows better results. In particular, for high CIR and corresponding low BER the fact that the noise is not white becomes significant. Here the h-based prefiltering has a performance loss of almost 5 dB compared to the x-based version.

4. Constant modulus algorithm

By construction, the MSE approach is restricted to the midamble where the transmitted symbols $s(n)$ are known. Another type of cost function was introduced by Kuzminskiy ([4], [5]). In the following, we will only consider the case where the feedback part of the prefilter unit is not present.

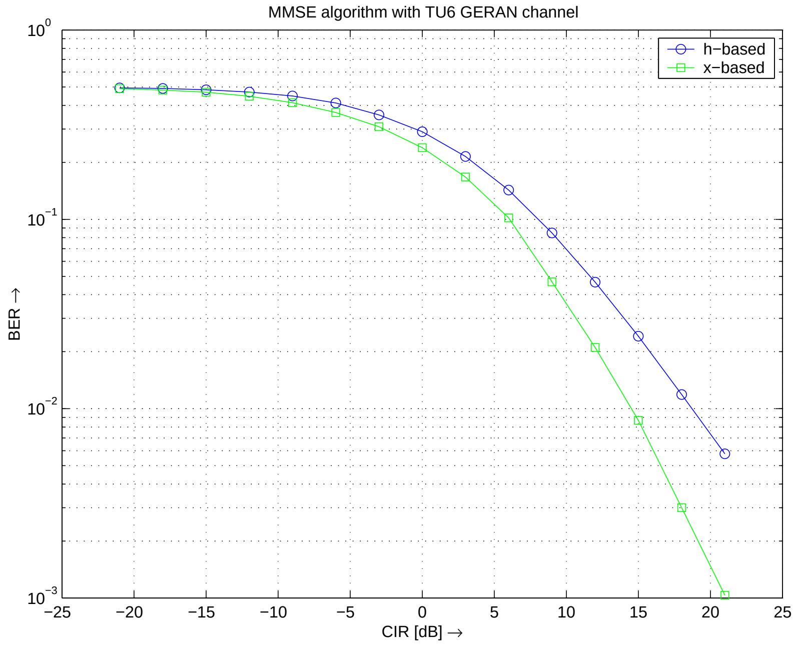

FIGURE 2. Comparison of h and x-based channel estimation

The cost function in [5] combines the MSE and CMA approach with a parameter ρ which gives the relative weight between these two components. The new cost function is defined by

$$J = \frac{1}{N_0} \sum_{n=n_0+1}^{n_0+N_0} J_{\mathrm{MSE}}(n) + \rho \, \frac{1}{N_1} \sum_{n=n_1+1}^{n_1+N_1} J_{\mathrm{CMA}}(n) \tag{8}$$

with

$$J_{\mathrm{MSE}}(n) = |\mathbf{p}^H \mathbf{x}(n) - s(n)|^2 \tag{9}$$

$$J_{\mathrm{CMA}}(n) = \left(|\mathbf{p}^H \mathbf{x}(n)| - 1\right)^2 \tag{10}$$

The ranges of the MSE and CMA algorithm are visualized in Figure 3. As the first sum in (8) equals the MSE part, it usually runs over the midamble (or some enlargement of it). The second part is the constant modulus approach. Note that a single term in the second sum vanishes if and only if the filtered sample $\mathbf{p}^H\mathbf{x}(n)$ at time n has modulus 1. By definition of the GSM (and also EDGE) symbols, the transmitted symbols lie on the unit circle. If all received symbols have the same modulus, then the effective channel impulse response has only one tap. So if this constant modulus condition holds after pre-filtering, we have equalized the channel up to one tap. Starting from the midamble to both directions, usually we will set $n_0 = n_1$.

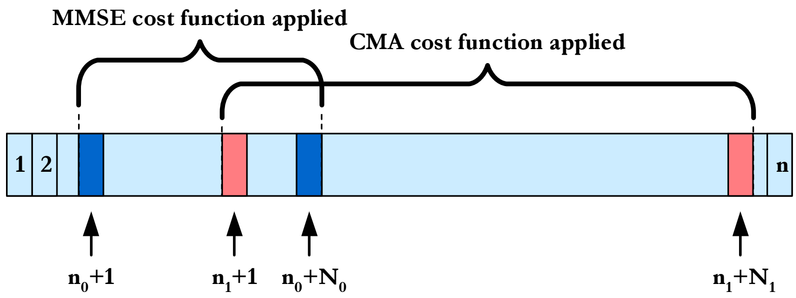

FIGURE 3. Cost function ranges for a joint MSE-CMA algorithm

In contrast to the MSE cost function, minimization of the CMA cost function has to be done iteratively. For this some variant of gradient search method should be applied.

The gradient of the combined cost function becomes

$$\frac{\partial}{\partial \mathbf{p}^H} J = \underbrace{\frac{1}{N_0}\left[\left(\sum_{n=n_0+1}^{n_0+N_0} \mathbf{x}(n)\mathbf{x}(n)^H\right)\mathbf{p} - \left(\sum_{n=n_0+1}^{n_0+N_0} \mathbf{x}(n)s(n)^*\right)\right]}_{\text{MSE part}} \tag{11}$$

$$+ \rho \cdot \underbrace{\frac{1}{N_1}\left[\sum_{n=n_1+1}^{n_1+N_1} \left(1 - |\mathbf{p}^H\mathbf{x}(n)|^{-1}\right)\mathbf{x}\mathbf{x}^H\right]\mathbf{p}}_{\text{CMA part}}.$$

The simple gradient search iteration then has the form

$$\mathbf{p}^{(n+1)} = \mathbf{p}^{(n)} - \mu \frac{\partial J}{\partial \mathbf{p}^H}. \tag{12}$$

Here μ is a real parameter called the stepsize of the iteration. Note that the performance of the gradient search iteration strongly depends on the choice of the stepsize.

As already mentioned, all CMA based cost functions can be seen as a blind method to estimate the interferer. For CIR values below or near 0 dB, all blind methods have the fundamental problem that they cannot distinguish between the desired user signal and the unwanted interferer signal. This can also be seen in the BER curves in Figure 4. Here for negative CIR values the combined MSE and CMA approach has no advantages. For high CIR values, however, the CMA part can improve the performance up to 1 dB and more. In the simulations for Figure 4, the parameter ρ was set to 1.

We end this section with some remarks. First, simulations show that the performance of the joint MSE-CMA approach is not very sensitive to the choice of the parameter ρ. We also want to stress that the performance gain of the CMA part is rather small and depends also on the choice of the stepsize μ in the gradient iteration. Due to these reasons, the CMA approach should be used carefully or with a more

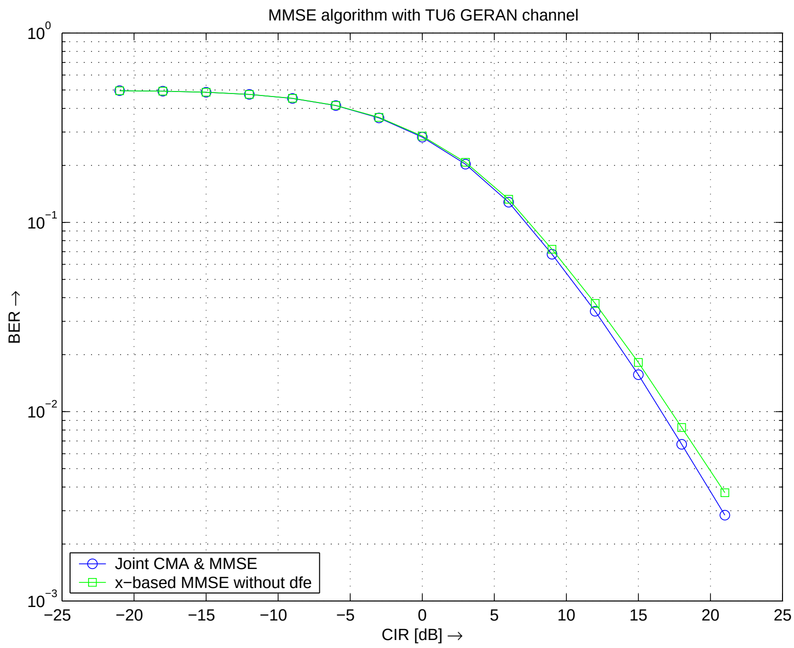

Figure 4. Comparison between MSE and joint MSE-CMA algorithm

sophisticated variant of the gradient search. Such variants are the Gauss-Newton iteration (see [5]), and versions of conjugate gradient (cg) methods.

We also want to remark that there are other possibilities to improve the performance of the pre-filtering structure in the presence of interferers. For instance, good performance improvements can be achieved when the input signal is oversampled, so we have several polyphases and use them in the pre-filtering structure. In this case one can speak of virtual antennas, and the resulting pre-filtering will be similar to a MIMO approach.

Another method uses an enlargement of the training sequence in the following sense: in the first iteration, the MSE cost function is based on the known midamble symbols. In a second iteration, we assume that the already decided symbols are correct. Thus, we can use them for the MSE approach in the same way as the midamble, enlarging the number of symbols which are the basis for the computation of the pre-filter coefficients. This iterative midamble enlargement gives significant performance improvement but also implies larger computational effort.

5. Conclusion

In the present note, we have considered the pre-filtering structure which is present in almost every GSM mobile station receiver. Pre-filtering is used to improve the signal's properties before equalization is done. Pre-filtering becomes even more important in the presence of an interferer as in this case the noise is no longer white.

The 'optimal' pre-filtering coefficients depend on the choice of the cost function. We discussed two main approaches: the MSE and the CMA approach. The MSE approach is based on the midamble where the transmitted symbols are known. Here we want to minimize the mean square error. There are two variants to address this minimization. In the first (h-based) we explicitly use an estimate for the channel impulse response. The second (x-based) is only based on the input samples of the actual burst.

Although the h-based version may have advantages concerning computational effort, it is not recommended in the presence of interferers. This is due to the fact that the stochastic assumptions behind the h-based coefficients are no longer satisfied. Therefore, it is no surprise to see that the x-based filter coefficients give significantly better performance.

The CMA approach can be seen as a variant of blind estimation and signal suppression. As the cost function does not use the correct (transmitted) symbol, all input samples of the GSM burst can be used to compute the CMA filter coefficients.

Simulations show that the CMA cost function has to be used very carefully. First, the cost function is no longer quadratic which leads to an iterative minimization procedure. Usually, such iteration is done in some variant of gradient search. This gives additional numerical difficulties, in particular the choice of the stepsize for the iteration. More sophisticated versions of gradient search can lead to high computational effort.

A good choice for the cost function is a combination of MSE and CMA. Compared to the MSE approach, we achieve a performance gain of about 1 dB for very high CIR. This can be explained as for high CIR the user signal can be clearly distinguished from the interferer signal, so the CMA cost function forces the user signal to be equalized.

The situation is different for low CIR (about 0 dB). Here the CMA method can no longer distinguish between user and interferer signal, and the joint MSE-CMA approach does not give better performance than the MSE method.

Summarizing, we can say that in an interferer scenario one should definitely use an x-based version of the MSE approach instead of an h-based version. An additional CMA part in the cost function may give better performance for high CIR but usually gives no improvement for low CIR. For the gradient search method which is part of the CMA algorithm, one has to be careful about the choice of the stepsize. A significantly better performance can possibly be obtained by more sophisticated gradient search methods with resulting larger computational effort. There are additional possibilities to improve the pre-filtering in the interferer case, e.g., the combination of several polyphases of the input signal.

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