

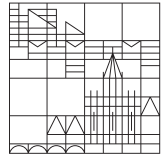
# Recent Advances in Real Algebra and Model Theory

## Birthday Workshop in honor of Salma Kuhlmann's scientific and career achievements

**Friday 23<sup>rd</sup> June 2023, 9:00 – 16:00**  
**in Room F426 and online**

<https://uni-konstanz-de.zoom.us/j/92031060582>  
(meeting ID: 92031060582)

**Abstract.** This workshop aims to celebrate Salma Kuhlmann's career and scientific achievements on the occasion of her birthday through contributions (in hybrid format) from her close collaborators, former and current students and postdocs. In particular, the workshop is meant to give a glimpse into her numerous research contributions to the mathematical community in Real Algebra and Model Theory as well as her strong empowerment of young researchers and especially of women in mathematics.



# Recent Advances in Real Algebra and Model Theory

## Schedule

### Conference Dinner

17:30

Good Rice

Salmansweilergasse 34  
78462 Konstanz

---

9:00 – 9:10 **Opening**  
*Brief Presentation on Salma's Career Path and Academic Contributions*

---

9:10 – 9:40 **Lasse Vogel**  
*Non-Abelian Left-Ordered Groups and the Archimedean Property*

9:50 – 10:20 **Didier Henrion** (online)  
*Solving Non-Linear Partial Differential Equations  
with the Infinite-Dimensional Moment-Sum-of-Squares Hierarchy*

---

10:20 – 10:50 Coffee Break

---

10:50 – 11:20 **Sarah Hess**  
*Sharu's Tale: A Narration by Salma Kuhlmann*

11:30 – 12:00 **Raul Curto** (online)  
*The Truncated Moment Problem for Unital Commutative Real Algebras*

---

12:00 – 13:30 Lunch Break

---

13:30 – 14:30 **Mickaël Matusinski**  
*About the Algebraic Closure of Multivariate Formal Power Series*

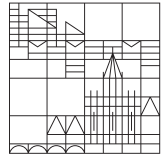
14:40 – 15:10 **Laura Wirth**  
*Archimedean Ordered Fields with the Independence Property*

15:20 – 15:50 **Tobias Kuna** (online)  
*An Intrinsic Characterization of Moment Functionals in the Compact Case*

---

17:30 Conference Dinner

---



# Recent Advances in Real Algebra and Model Theory

## Abstracts

Lasse Vogel, University of Konstanz

### **Non-Abelian Left-Ordered Groups and the Archimedean Property**

**Abstract.** Typically when discussing ordered groups we consider groups with a strict ordering that is compatible with the group's multiplication from either side. A theorem by Hölder now states that any Archimedean ordered group is already Abelian. Archimedean can be defined classically as "for every two positive non-identities, some power of the one will be larger than the other" or in this setting equivalently as "there is no proper non-trivial convex subgroup". If we now consider the weaker case of left-ordered groups, i.e. we don't assume compatibility with right-multiplication, then this result falls apart. In this talk I will give a short exposition on which properties are lost in this setting and an example for a non-Abelian group without proper convex subgroup.

Didier Henrion, LAAS-CNRS Toulouse and Czech Tech. University Prague

### **Solving Non-Linear Partial Differential Equations with the Infinite-Dimensional Moment-Sum-of-Squares Hierarchy**

**Abstract.** We show how the recent fundamental contributions by Salma and colleagues on the moment problem on nuclear spaces are instrumental to solving a broad class of non-linear partial differential equations with the moment-sum-of-squares hierarchy, up to its numerical implementation with semidefinite optimization.

Sarah Hess, University of Konstanz

### **Sharu's Tale: A Narration by Salma Kuhlmann**

**Abstract.** The cone  $\mathcal{P}_{n+1,2d}$  of all positive semidefinite real forms in  $n+1$  variables of degree  $2d$  contains the cone  $\Sigma_{n+1,2d}$  of all real forms that are representable as finite sums of squares of real forms. Hilbert proved in 1888 that  $\Sigma_{n+1,2d} = \mathcal{P}_{n+1,2d}$  if and only if  $n+1 = 2$  or  $2d = 2$  or  $(n+1, 2d) = (3, 4)$ .

In this talk, we outline several new takes on this result, starting off by stating the Choi-Lam analogue of Hilbert's 1888 Theorem for symmetric forms. We highlight contributions made by Goel, Kuhlmann and Reznick to this result and moreover discuss their analogue for even symmetric form. As a spin-off to this project, Goel and Kuhlmann developed a construction of intermediate cones between  $\Sigma_{n+1,2d}$  and  $\mathcal{P}_{n+1,2d}$  via a Gram matrix approach. Based on this construction, we induce a filtration of intermediate cones between  $\Sigma_{n+1,2d}$  and  $\mathcal{P}_{n+1,2d}$  along a filtration of irreducible projective varieties containing the Veronese variety and investigate it for proper inclusion. This eventually allows us to give a generalization of Hilbert's 1888 Theorem.

Raul Curto, University of Iowa

### **The Truncated Moment Problem for Unital Commutative Real Algebras**

**Abstract.** Let  $A$  be a unital commutative  $\mathbb{R}$ -algebra,  $K$  a closed subset of the character space of  $A$ , and  $B$  a linear subspace of  $A$ . For a linear functional  $L : B \rightarrow \mathbb{R}$ , we investigate conditions under which  $L$  admits an integral representation with respect to a positive Radon measure supported in  $K$ . When  $A$  is equipped with a submultiplicative seminorm, we employ techniques from the theory of positive extensions of linear functionals to prove a criterion for the existence of such an integral representation for  $L$ .

When no topology is prescribed on  $A$ , we identify suitable assumptions on  $A$ ,  $K$ ,  $B$  and  $L$  which allow us to construct a seminormed structure on  $A$ , so as to exploit our previous result to get an integral representation for  $L$ . Our main theorems allow us to extend some well-known results on the Classical Truncated Moment Problem, the Truncated Moment Problem for point processes, and the Subnormal Completion Problem for 2-variable weighted shifts.

We also analyze the relation between the Full and the Truncated Moment Problem in our general setting; we obtain a suitable generalization of Stochel's Theorem, which readily applies to Full Moment Problems for localized algebras.

The talk is based on joint work with Mehdi Ghasemi, Maria Infusino and Salma Kuhlmann.

Mickaël Matusinski, University of Bordeaux

### **About the Algebraic Closure of Multivariate Formal Power Series**

**Abstract.** Let  $K$  be a field of characteristic zero and  $x = (x_1, \dots, x_r)$ . We consider the algebraic closure of  $K[[x]]$  as a subfield of the so-called field of rational polyhedral Puiseux series, and call its elements algebroid Puiseux series. We deal with the two following problems:

- given a polynomial equation  $P(x, y) = 0$  for  $P \in K[[x]][y]$ , provide a closed form formula for the coefficients of an algebroid Puiseux series solution  $y(x)$  in terms of the coefficients of  $P$ ;
- given an algebroid Puiseux series  $y(x)$ , reconstruct algorithmically the coefficients of a vanishing polynomial  $P \in K[[x]][y]$  using the coefficients of the series.

Our strategy involves the answers that we recently obtained to the same type of questions about algebraic Puiseux series, i.e. for the algebraic closure of  $K(x)$ .

Note that this field of algebraic series contains the field of fractions of  $K[[x]]$  which was recently studied by Sebastian, Salma and Michele in the more general context of Hahn power series. I will discuss some of the questions they raise.

This is about a joint work in progress with Michel HICKEL (U. of Bordeaux).

Laura Wirth, University of Konstanz

## Archimedean Ordered Fields with the Independence Property

**Abstract.** The aim of this talk is to systematically study archimedean ordered fields with the independence property. Our investigations are motivated by a conjecture on the classification of infinite fields without the independence property, which goes back to Shelah [5], [6]. With time, this conjecture has evolved, was modified and was also specialized for the case of ordered fields (cf. [1], [2], [3], [4]). We conjecture that any *archimedean* ordered field without the independence property is real closed. We exhibit an approach for verifying the independence property and present several examples of fields that we are considering. The necessary notions, including the independence property, will be introduced.

### References:

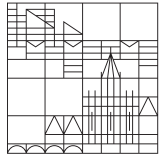
- [1] K. DUPONT, A. HASSON and S. KUHLMANN, 'Definable valuations induced by multiplicative subgroups and NIP fields', *Arch. Math. Logic* **58** (2019) 819–839.
- [2] Y. HALEVI, A. HASSON and F. JAHNKE, 'A conjectural classification of strongly dependent fields', *Bull. Symb. Log.* **25** (2019) 182–195.
- [3] Y. HALEVI, A. HASSON and F. JAHNKE, 'Definable  $V$ -topologies, Henselianity and NIP', *J. Math. Log.* **20** (2020)
- [4] L. S. KRAPP, S. KUHLMANN and G. LEHÉRICY, 'Strongly NIP almost real closed fields', *MLQ Math. Log. Q.* **67** (2021), 321–328.
- [5] S. SHELAH, 'Stability, the f.c.p., and superstability; model theoretic properties of formulas in first order theory', *Ann. Math. Logic* **3** (1971) 271–362.
- [6] S. SHELAH, 'Strongly dependent theories', *Isr. J. Math.* **204** (2014) 1–83.

Tobias Kuna, University of L'Aquila

## An Intrinsic Characterization of Moment Functionals in the Compact Case

**Abstract.** We discuss characterizations of linear functionals  $L$  on an unital commutative real algebra  $A$  which can be represented as integral w.r.t. a compactly supported Radon measure on the character space of  $A$ . We give a characterization by the following three types of conditions: bounds on the growth of  $(L(a^n))_n$ , non-negativity of  $L$  on Archimedean quadratic models, and continuity of  $L$  w.r.t. submultiplicative seminorms on  $A$ . We will relate each of these conditions to a different technique solving this instance of the moment problem. Surprisingly, we can also provide an exact characterization of the compact support of the representing Radon measure purely in terms of  $L$ .

This is a joint work with Maria Infusino, Salma Kuhlmann and Patrick Michalski.



# Recent Advances in Real Algebra and Model Theory

## Birthday Wishes

Happy Birthday, Salma, from the prairie of Illinois, and best wishes for many more!

*Bruce*

---

Dear Salma: there is statistical evidence that birthdays are excellent for your health: Indeed, people with many birthdays live longer than others :) So what a great day to celebrate your birthday! I send you (probably) the northern most birthday wishes today and which you that this new year may bring you lots of happiness, success and many beautiful moments with your family and friends!

*Cordian*

---

Joyeux anniversaire

*Didier*

---

Dear Salma, I wish you all the best for your birthday. May you enjoy many more years of playful exploration in the delightful world of valuations and orders.

*Gabriel*

---

Happy birthday to a wonderful supervisor, a great professor and an impressive role model for women in mathematics! Many thanks for your patient guidance as well as your valuable advice and comments in any regard!

*Laura*

---

Dear Salma, I wish you all the best for your special day. Sorry that I will only be virtually celebrating with you, but I hope you have a great day, lots of fun, and interesting talks. I can't believe that we only met rather recently. Thank you so much for allowing me to co-supervise my very first PhD student with you. I am sure this period will always be incredibly special to me. I learned a lot from you these last years, e.g., how to be a good mentor and supervisor. Cannot wait to see you again and toast to you in person. Best

and cheers,

*Mareike*

---

Very often your “Learn to say no” rings in my ears and reminds me your inspiring determination and limitless energy. Thanks for this and many other advices, thanks for your honest friendship and your constant support along my career and personal path.

Wishing you love, health and happiness, I send you my warmest hug in this special birthday celebration!

*Maria*

---

My best and most sincere wishes of further happiness, love and fulfillment! Thank you so much, for all that you taught me both about maths and about myself and, most of all, for believing in me when even I did not.

*michele*

---

Bel et heureux anniversaire!

Je suis heureux de toutes ces années partagées et d’imaginer celles à venir.

*Mickaël*

---

Thank you for being an inspirational and thoughtful professor and supervisor. I have always had a passion for math, but you showed me what it is like to be a mathematician and to take care of the tiniest details without losing sight of the big picture. Happy Birthday!

*Moritz*

---

Dear Salma, Heartfelt Congratulations on the occasion of your 65th birthday! Thank you for our friendship, our professional interactions, and all the pearls of wisdom I have collected along the way. I have learned a lot from our collaborations, and I have seen first-hand how you and your mentoring positively impact the life and career of your mentees; you’re wonderful to be around! Keep up the great work! I will see you soon, in person, in Eindhoven! Warm regards,

*Raul*

---

Happy birthday to you, Salma, a great person who means so many different things to all of us - guiding supervisor, valuable collaborator, loyal mentor...trusted friend! I wish you all the best for the years to come and plenty of exciting beautiful math to discover. Best,

*Sarah*

---

Happy birthday to a wonderful supervisor, mentor and adviser! Thank you for your constant support und continuous guidance, for the past 8 years and more to come.

*Sebastian*